The improvement of data selection method in data-driven model predictive control of uncertain systems

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***Abstract*—Data quality is pivotal for the performance of data-driven model predictive control systems. While traditional data processing methods, such as cleaning and enrichment, are necessary, they do not always improve the controller performance. To tackle this problem, this paper introduces a new data selection method specifically designed for data-driven model predictive control. It includes a controller performance evaluation approach that assesses the quality of the current data, enabling the controller to select a set of informative data elements that enhance its performance. Additionally, a sliding window mechanism is implemented to compare current data against historical data, preventing the loss of significant patterns or trends. A simulation example illustrates the effectiveness of the proposed data selection method.**

***Keywords*—Data selection, controller performance evaluation, data-driven, model predictive control.**

I. Introduction

Model predictive control(MPC) is one of the most prevalent advanced control strategies employed in today's process industry, such as robotics [1], crude distillation [2] and automotive driving [3]. It obtains better control performance while satisfying the system constraints.

Traditional MPC provides stability against minor model mismatch and disturbance. Yet, as these mismatches or disturbances grow, the closed-loop system can become unstable, potentially leading to divergence of the state. To tackle the difficulty of managing systems with constraints under uncertain model parameters, the concept of robust MPC has been introduced [4]. This approach, while enhancing robustness, often introduces a level of conservatism that affects the control performance [5].

To address the conservatism inherent in robust MPC, researchers have incorporated data into the model update process or controller design. Historical data, rich with dynamic information about unknown models, can be harnessed to estimate models for systems with undisclosed parameters. This adaptive MPC technique has been extensively implemented in industrial settings, yielding positive feedback in terms of control effectiveness [3]. However, the online identification process does require a significant allocation of computational resources.

Another type of method bypasses model identification, using data to design the MPC controller directly. It is simple and effective. Data-driven MPC strategies are categorized as offline or online, depending on the data source. In industrial systems, the dynamic operating points of the equipment may not be completely represented by offline data. In contrast, online data refreshment enables the system to adapt to the system's changing dynamics in real time, which is why online data-driven MPC is increasingly favored [6].

An online data-driven input-mapping method is proposed in [7], which uses a linear combination of historical input data and an additional free input to determine the current control input. In [8], a data selection strategy is proposed, aimed at filtering linearly independent data vectors to minimize the data required to predict the system's future trajectory. Meanwhile, [9] presents a similarity index for assessing the resemblance between current and previously stored past data. Despite these advancements, the controller's performance was not factored into the data selection process. Nevertheless, the performance of data-driven MPC is greatly influenced by the quality of the prior knowledge database as well as its quantity[10].

At present, there are many research results on controller performance evaluation, such as minimum variance control benchmark [11], linear quadratic Gaussian benchmark [12], and generalized minimum variance control law [13]. However, these methods all depend on predefined models. In [14], a period of reference data from the process with satisfactory control performance is used as a benchmark. The key limitation of this method, however, is that it relies on prior knowledge for selecting the reference data. As far as we are aware, there is currently no research that integrates online data-driven data selection with controller performance evaluation.

In this paper, we design a data selection method for uncertain systems within the data-driven MPC strategy, which does not rely on certain models. We utilize process data to evaluate data quality, employing the ratio of differences between the current and previous loss functions to assess whether the current data enhances controller performance. The approach from [9] focuses solely on comparing current data to historical data, which may lead to the omission of significant data and the loss of critical patterns or trends. To address this, we incorporate a sliding window strategy, commonly employed in data stream mining [15]. By implementing this, we maintain a collection of high-quality data items over a period, ensuring that optimal data sets are identified and that valuable data is not systematically excluded.

The rest of this paper is organized as follows. The problem formulation is presented in Section 2. In Section 3, a novel data selection method for the data-driven MPC strategy is proposed. Section 4 provides an example to demonstrate the efficacy of the methods proposed in this paper. Finally, Section 5 presents key conclusions drawn from the study.

Notation: Let $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{N}$ denote the field of real numbers, n-dimensional real space, and the set of non-negative integers, respectively. Let $\mathbb{N}_{[a,b]} = \{a, a+1, \cdots, b\}$, for $a, b \in \mathbb{N}$. Let $\mathbb{N}_{\geq a} = \{a, a+1, \cdots\}$, for $a \in \mathbb{N}$. $Co\{\cdot\}$ denotes the convex hull. Given a symmetric matrix P, $P \succ 0$ means that the matrix P is positive definite. For a column vector x and a matrix $P \succ 0$ with appropriate dimensions, $\|x\|_P^2 = x^T P x$. For column vectors $x_1, \cdots, x_n$, $\text{col}[x_1, \cdots, x_n] = [x_1^T, \cdots, x_n^T]^T$. $\|\cdot\|_2$ denotes 2-norm.

II. Problem formulation

Consider the discrete-time uncertain system as follows

$$x(k+1) = Ax(k) + Bu(k) + d(k), \tag{1}$$

and the system constraints satisfy

$$Fx(k) + Gu(k) \leq 1, \tag{2}$$

where $x(k) \in \mathbb{R}^{n_x}$, $u(k) \in \mathbb{R}^{n_u}$, and $d(k) \in \mathbb{R}^{n_x}$ represent the state, the control input, and the disturbance, respectively. A and B are both unknown matrices, which satisfy the following assumption.

Assumption 2.1: The system matrices satisfy $A = A_0 + \Delta_A$, $B = B_0 + \Delta_B$, where $[\Delta_A, \Delta_B] \in \Xi_\Delta \triangleq C_O\{[\Delta_A^{(1)}, \Delta_B^{(1)}], \cdots, [\Delta_A^{(I)}, \Delta_B^{(I)}]\}$.

A_0 and B_0 are certain known matrices which can be called as the nominal model. The uncertainties Δ_A and Δ_B can be represented by a convex set of L vertices $\{[\Delta_A^{(1)}, \Delta_B^{(1)}], ..., [\Delta_A^{(I)}, \Delta_B^{(I)}]\}$. We define $[A^{(i)}, B^{(i)}] \triangleq [A_0 + \Delta_A^{(i)}, B_0 + \Delta_B^{(i)}]$, $i \in \mathbb{N}_{[1,I]}$, then the model $[A, B]$ is in the collection of Ξ, i.e.,

$$[A, B] \in \Xi \triangleq C_0\{[A^{(1)}, B^{(1)}], \cdots, [A^{(I)}, B^{(I)}]\}. \tag{3}$$

Assuming the state of system (1) is completely measurable, the aim of this paper is to develop a data-driven MPC controller that integrates an online data selection method. This method is pivotal for the identification of informative data, thereby significantly improving the controller's performance and ensuring the effective implementation of the MPC strategy.

At time k, given the data items at past M moments generated by the unknown system $\sum_{p=1}^{M} x(k-p), \sum_{p=1}^{M} u(k-p)$ and the current state $x(k)$. Define the matrices that collect these data

$$X_k^- := [x(k-1), x(k-2), \cdots, x(k-M)], \tag{4}$$
$$U_k^- := [u(k-1), u(k-2), \cdots, u(k-M)], \tag{5}$$
$$X_k^+ := [x(k), x(k-1), \cdots, x(k-M+1)]. \tag{6}$$

Given that the current state and input satisfies $\{x(k), u(k)\} = \{\sum_{p=1}^{M} \beta_p x(k-p), \sum_{p=1}^{M} \beta_p u(k-p)\}$, then the next step perdicted state $x_{(1|k)} = \sum_{p=1}^{M} \beta_p (x(k-p+1) - d(k-p)) + d(k)$.

III. Input-mapping MPC with a data selection method

In this section, we propose an input-mapping based data-driven MPC strategy with a data selection method, aimed at improving the control performance of system (1).

A. Input-mapping MPC

Input-mapping methods utilize historical states and inputs to represent current and future multi-step control states and inputs. As analyzed above, the future state $x_{(\tau|k)}$ can be composed of the past state information $\gamma_{(\tau|k)}$ and state residual $\delta_{(\tau|k)}$,

$$x_{(\tau|k)} = \gamma_{(\tau|k)} + \delta_{(\tau|k)} + w_{(\tau|k)}, \tag{7}$$

The linear combination of historical states $\gamma_{(\tau|k)} \in \mathbb{R}^{n_x}$:

$$\gamma_{(\tau|k)} = \sum_{p=1}^{M-p} (l_{(\tau|k)})_p x(k-p), \tau \in \mathbb{N}_{[0,N_P-1]}, \tag{8}$$

Similarly, the future input $u_{(\tau|k)}$ consists of the past input information $\upsilon_{(\tau|k)}$ and input residual $\sigma_{(\tau|k)}$. In addition, we use the dual-mode MPC strategy [16].

$$u_{(\tau|k)} = \begin{cases} \upsilon_{(\tau|k)} + \sigma_{(\tau|k)}, & \tau \in \mathbb{N}_{[0,N_P-1]}, \\ Kx_{(\tau|k)}, & \tau \in \mathbb{N}_{\mathbb{P} \geq N}, \end{cases} \tag{9}$$

where the stabilizing feedback gain K enables the closed-loop system matrix $\phi^{(j)} = A^{(j)} + B^{(j)}K$ to satisfy the Schur stability. The linear combination of historical inputs $\upsilon_{(\tau|k)} \in \mathbb{R}^{n_u}$ can be represented as:

$$\upsilon_{(\tau|k)} = \sum_{p=1}^{M-p} (l_{(\tau|k)})_p u(k-p), \tau \in \mathbb{N}_{[0,N_P-1]}, \tag{10}$$

where the linear combination coefficient $l_{(\tau|k)} \in \mathbb{R}^{N_P-\tau}$ is the online optimization variable. The input residual $\sigma_{(\tau|k)} \in \mathbb{R}^{n_u}$ can be represented as

$$\sigma_{(\tau|k)} = K(\delta_{(\tau|k)} + w_{(\tau|k)}) + \varrho_{(\tau|k)}, \tau \in \mathbb{N}_{\geq 0}, \tag{11}$$

where $\varrho_{(\tau|k)}$ is the online optimization variable.

On the basis of equation (9) and (7), the predicted state can be represented as

$$\begin{aligned} x_{(\tau+1|k)} = {} & \gamma_{(\tau+1|k)} + H_{(\tau|k)} + (A+BK)\delta_{(\tau|k)} \\ & + B\sigma_{(\tau|k)} + w_{(\tau+1|k)}, \end{aligned} \tag{12}$$

where $w_{(\tau+1|k)} = -\sum_{p=1}^{M-\tau}[l_{(\tau|k)}]_p d(k-\tau) + (A+BK)w_{(\tau|k)} + d_{(\tau|k)}$.

The state residual $\delta_{(\tau|k)} \in \mathbb{R}^{n_x}$ can be represented as

$$\begin{aligned}\delta_{(\tau|k)} = H_{(\tau-1|k)} + (A+BK)\sigma_{(\tau-1|k)} \\ + Bc_{(\tau-1|k)}, \tau \in \mathbb{N}_{\geq 1},\end{aligned} \tag{13}$$

where $\tau_{(0|k)} = x_{(0|k)} - \gamma_{(0|k)}$. $H_{(\tau-1|k)}$ is defined as

$$\begin{aligned}&H_{(\tau-1|k)} \\ &\triangleq \sum_{p=2}^{M-\tau+1}\left[\left(l_{(\tau-1|k)}\right)_p - \left(l_{(\tau|k)}\right)_{p-1}\right]x(k-p+1) \\ &+ \left(l_{(\tau-1|k)}\right)_1 x(k), \tau \in \mathbb{N}_{[1,N_P-1]},\end{aligned} \tag{14}$$

$$\begin{aligned}&H_{(N_P-1|k)} \triangleq \sum_{p=1}^{M-N_P+1}\left(l_{(N_P-1|k)}\right)_p x(k-p+1), \\ &H_{(\tau|k)} = 0, \tau \in \mathbb{N}_{\geq N}.\end{aligned} \tag{15}$$

The input-mapping data-driven robust MPC aims to calculate control inputs by solving linearly constrained quadratic programming problems at each moment. Some symbols are introduced here to define the objective function,

$$\begin{aligned}&\underline{l}_{(\tau|k)} = \mathrm{col}\left[l_{(\tau|k)}, l_{(\tau+1|k)}, \cdots, l(\tau+N_P-1|k)\right], \\ &\underline{\gamma}_{(\tau|k)} = \mathrm{col}\left[\gamma_{(\tau|k)}, \gamma_{(\tau+1|k)}, \cdots, \gamma(\tau+N_P-1|k)\right], \\ &\underline{\upsilon}_{(\tau|k)} = \mathrm{col}\left[\upsilon_{(\tau|k)}, \upsilon_{(\tau+1|k)}, \cdots, \upsilon(\tau+N_P-1|k)\right], \\ &\underline{\sigma}_{(\tau|k)} = \mathrm{col}\left[\sigma_{(\tau|k)}, \sigma_{(\tau+1|k)}, \cdots, \sigma(\tau+N_P-1|k)\right], \\ &\underline{H}_{(\tau|k)} = \mathrm{col}\left[H_{(\tau|k)}, H_{(\tau+1|k)}, \cdots, H(\tau+N_P-1|k)\right], \\ &\theta_{(\tau|k)} = \mathrm{col}\left[\underline{\gamma}_{(\tau|k)}, \underline{\upsilon}_{(\tau|k)}, \delta_{(\tau|k)}, \underline{\sigma}_{(\tau|k)}, \underline{H}_{(\tau|k)}\right].\end{aligned}$$

The matrices $V_{n_x} \in \mathbb{R}^{n_x \times Nn_x}, V_{n_u} \in \mathbb{R}^{n_u \times Nn_u}$ are introduced as follows:

$$\begin{aligned}V_{n_x} = \left[\begin{array}{cccc} I_{n_x} & 0 & \cdots & 0 \end{array}\right], \\ V_{n_u} = \left[\begin{array}{cccc} I_{n_u} & 0 & \cdots & 0 \end{array}\right].\end{aligned}$$

From these definitions, we can derive the relationships that $\gamma_{(\tau|k)} = V_{n_x}\underline{\gamma}_{(\tau|k)}, \upsilon_{(\tau|k)} = V_{n_u}\underline{\upsilon}_{(\tau|k)}, \sigma_{(\tau|k)} = V_{n_u}\underline{\sigma}_{(\tau|k)}, H_{(\tau|k)} = V_{n_x}\underline{h}_{(\tau|k)}$. Furthermore, we introduce the transfer matrices $W_{n_x} \in \mathbb{R}^{Nn_x \times Nn_x}, W_{n_u} \in \mathbb{R}^{Nn_u \times Nn_u}$ as follows:

$$\begin{aligned}[W_{n_x}]_{\tau,k} = \begin{cases} I_n, k = \tau+1, \\ \mathbf{0}, \quad k \neq \tau+1, \end{cases} \\ [W_{n_u}]_{\tau,k} = \begin{cases} I_m, k = \tau+1, \\ \mathbf{0}, \quad k \neq \tau+1. \end{cases}\end{aligned}$$

Similarly, we can derive the relationships that $\gamma_{(\tau+1|k)} = V_{n_x}\underline{\gamma}_{(\tau|k)}, \upsilon_{(\tau+1|k)} = V_{n_u}\underline{\upsilon}_{(\tau|k)}, \sigma_{(\tau+1|k)} = V_{n_u}\underline{\sigma}_{(\tau|k)}, H_{(\tau+1|k)} = V_{n_x}\underline{H}_{(\tau|k)}$. The optimization problem in the input-mapping data-driven robust MPC algorithm is described in detail as follows

$$\begin{aligned}\min_{\{\sigma_{(\tau|k)}, l_{(\tau|k)}\}_{\tau \in \mathbb{N}_{[0,N_P-1]}}} \quad & \lambda(k)^T N(k)\lambda(k) \\ s.t. \quad & Fx(k) + Gu(k) \leq 1.\end{aligned} \tag{16}$$

$N(k)$ is a positive definite matrix that can be represented $N(k) = D(k)^T P D(k)$, where $D(k)$ satisfies $\lambda(k) = D(k)\theta_{(0|k)}$ and P satisfies $P \geq (\zeta^{(j)})^T P \zeta^{(j)} + \bar{Q}, \forall j \in \mathbb{N}_{[1,L]}$. $\zeta^{(j)}$ and $\bar{Q}$ can be further represented as $\zeta^{(j)} = \mathrm{diag}(W_{n_x}, W_{n_u}, \tilde{\zeta}^{(j)})$, $\bar{Q} = E_Q^T \mathrm{diag}(Q, R) E_Q$, where $\tilde{\zeta}^{(j)}, E_Q$ are defined as follows:

$$\begin{aligned}\tilde{\zeta}^{(j)} &= \left[\begin{array}{ccccc} \phi^{(j)} & B^{(j)} & 0 & V_{n_x} & 0 \\ 0 & W_{n_u} & & 0 & 0 \\ 0 & 0 & 0 & W_{n_x} & \end{array}\right], \\ E_Q &= \begin{bmatrix} V_{n_x} & \mathbf{0} & I_{n_x} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & V_{n_u} & K & V_{n_u} & \mathbf{0} \end{bmatrix}.\end{aligned}$$

$\lambda(k)$ is defined as

$$\lambda(k) = \mathrm{col}[\underline{l}_{(0|k)}, 1, \underline{\sigma}_{(0|k)}]. \tag{17}$$

B. Data selection strategy

This subsection proposes a data selection method to sort out data items in data-driven MPC. Due to the presence of disturbances within the system, the data near the origin does not accurately reflect the system's dynamic characteristics. To address this, we have established a threshold value for the state, denoted as δ_x, below which data points will be excluded from the data set. In addition, based on the method checking the similarity index [9], this method adds a controller performance evaluation benchmark and sliding window technology.

A linear quadratic Gaussian benchmark is proposed in [12] to assess controller performance. Nonetheless, its applicability may be limited in systems with no constraints. To assess the controller performance, we consider the ratio of the change in the loss function from one moment to the next. Specifically, the performance metric is defined by the quotient of the loss function's differences between the current and preceding instances. Define $n_{(\tau|k)} = \|x_{(\tau|k)}\|_Q^2 + \|u_{(\tau|k)}\|_R^2$. The controller performance at the previous time $k-1$ and the current time k can be represented as follows, respectively:

$$J_\infty(k-1) = \sum_{\tau=0}^{\infty}(n_{(\tau|k-1)}, J_\infty(k) = \sum_{\tau=0}^{\infty}(n_{(\tau|k)}). \tag{18}$$

Thus, define the data-driven controller performance benchmark as

$$\begin{aligned}\eta_k &= \frac{J_\infty(k-1) - J_\infty(k)}{n_{(0|k-1)}} \\ &= \frac{n_{(0|k-1)} - n_{(0|k)}}{n_{(0|k-1)}}.\end{aligned} \tag{19}$$

This benchmark delineates the rate of reduction in the loss function subsequent to the introduction of a control input at time k. An elevated benchmark value is indicative of a more rapid descent of the loss function, which in turn, correlates with an enhanced controller performance.

The controller performance benchmark operates by comparing current data solely against a dataset of preceding historical data, thereby determining the retention or exclusion of the data. Nonetheless, such an approach risks neglecting significant patterns or emerging trends. To address this limitation, we introduce a sliding window methodology. This technique is particularly effective in discerning the most exemplary data points within a designated timeframe, thereby averting the gradual loss of data integrity. The sliding window's scope is characterized by its length, represented as N_S. The historical data repository is bifurcated into two segments: one segment is a static component of historical data, while the other is a dynamic component that is periodically refreshed in tandem with the sliding window's progression.

To elucidate our method more clearly, Figure 1 presents a schematic diagram showing the logical flow and interactions of each step. N_T represents the length of the static component of the data. The comprehensive algorithm of the proposed method is encapsulated in Algorithm 1.

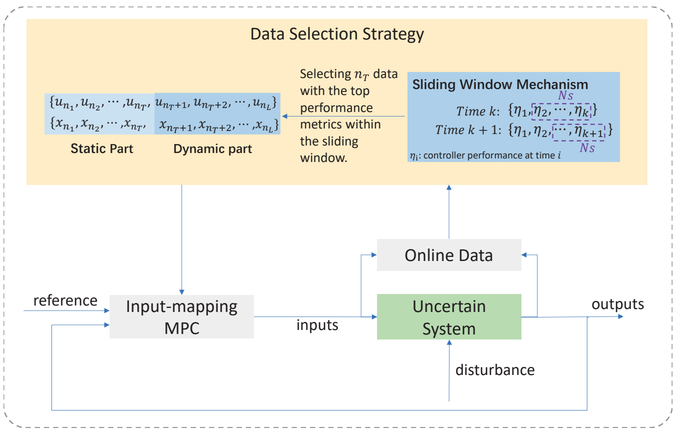

Fig. 1. The diagram of input-mapping MPC strategy with data selection method.

Theorem 3.1: (Recursive feasibility and stability) When system (1) satisfies Assumption 2.1, Algorithm 1 has recursive feasibility and closed-loop stability.

Proof: Assume problem (16) can be solved at k. Define the optimal solution sequence $\sigma^*_{(0|k)} = \{\sigma^*_{(0|k)}, \cdots, \sigma^*_{(N_P-1|k)}\}$, $l^*_{(0|k)} = \{l^*_{(0|k)}, \cdots, l^*_{(N_P-1|k)}\}$. Thus the optimal historical state and input linear combination variables as $\gamma^*(k) = \{\gamma^*_{(0|k)}, \cdots, \gamma^*{}_{(N_P-1|k)}\}$, $\upsilon^*(k) = \{\upsilon^*_{(0|k)}, \cdots, \upsilon^*_{(N_P-1|k)}\}$, the optimal auxiliary variables is $H^*(k) = \{H^*_{(0|k)}, \cdots, H^*_{(N_P-1|k)}\}$. Define the optimal state residual set $\delta^*_{(\tau|k)}$ and input residual set $\sigma^*_{(\tau|k)}$

$$\delta^*_{(\tau|k)} \in \Delta^*_{(\tau|k)} = Co\{\delta^{(\tilde{i}_{1:\tau})*}_{(\tau|k)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}$$
$$\sigma^*_{(\tau|k)} \in \Sigma^*_{(\tau|k)} = Co\{\sigma^{(\tilde{i}_{1:\tau})*}_{(\tau|k)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}$$

By using the optimal solution at time k, the solution sequence and linear combination coefficient for time $k+1$ can be constructed as:

$$\begin{aligned}\hat{\sigma}(k+1) =& \{\sigma^*_{(1|k)}, \sigma^*_{(2|k)}, \cdots, \sigma^*_{(N_P-1|k)}, \mathbf{0}\},\\ \hat{l}(k+1) =& \{l^*{}_{(0|k+1)}, l^*{}_{(1|k+1)}, \cdots, l^*{}_{(N_P-2|k+1)}, \mathbf{0}\},\end{aligned} \tag{20}$$

Algorithm 1: Input-mapping MPC strategy with data selection method.

Input: Select a robust and stabilizing feedback gain K, identify a robust positive invariant set Ω, specify the length of historical data N_L, and define the sliding window duration N_s;

for $k = 1, 2, \ldots$ **do**
- Solve the optimization problems (16) and apply the control inputs (11) to the system;
- Obtain the system state $x(k+1)$ and calculate the performance metric η_k for the current time step k (19);
- Store the current control performance metric value η_k at the present instant k;
- **if** $\|x\|_2$ exceeds δ_x **then**
  - **if** $k <= N_S$ **then**
    - Identify the top N_T data points based on the highest η_k values;
    - Fix N_T data in the data sets X^-_k,U^-_k,X^+_k;
  - **else**
    - Determine the most optimal $N_L - N_T$ data points exhibiting the highest η_k values;
    - Replace the dynamic elements with the selected data points;
  - **end**
- **end**
- Set $k := k+1$;

end

According to (10), (12) and (13), it can be obtained:

$$\begin{aligned}\hat{\gamma}_{(\tau|k+1)} &= \gamma^*_{(\tau+1|k)}, \tau \in \mathbb{N}_{[0,N_P-1]},\\ \hat{\gamma}_{(\tau|k+1)} &= \mathbf{0}, \tau \in \mathbb{N}_{\geq N_P},\\ \hat{\upsilon}_{(\tau|k+1)} &= \upsilon^*_{(\tau+1|k)}, \tau \in \mathbb{N}_{[0,N_P-1]},\\ \hat{\upsilon}_{(\tau|k+1)} &= \mathbf{0}, \tau \in \mathbb{N}_{\geq N_P},\\ \hat{H}_{(\tau|k+1)} &= H^*_{(\tau+1|k)}, \tau \in \mathbb{N}_{[0,N_P-1]},\\ \hat{H}_{(\tau|k+1)} &= \mathbf{0}, \tau \in \mathbb{N}_{\geq N_P}.\end{aligned} \tag{21}$$

The state residual set $\hat{\Delta}_{(\tau|k+1)}$ and the optimal state set $\hat{\Sigma}_{(\tau|k+1)}$ can be constructed as:

$$\hat{\Delta}_{(\tau|k+1)} = Co\{\hat{\delta}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}, \tag{22}$$
$$\hat{\Sigma}_{(\tau|k+1)} = Co\{\hat{\sigma}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}. \tag{23}$$

constructing the vertices as follows:

$$\hat{\delta}_{(0|k+1)} = \sum_{i=1}^{I} h^{(i)}(\delta^{(i)*}_{(1|k)} + w^{(i)*}_{(1|k)}),$$
$$\hat{\delta}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)} = \sum_{i=1}^{L} h^{(i)}\delta^{(i,\tilde{i}_{1:\tau})*}_{(\tau+1|k)} + \phi^{(i_\tau)}\cdots\phi^{(i_1)}w^*_{(1|k)}, \tau \in \mathbb{N}_{[1,N_P-1]},$$
$$\hat{\delta}^{(\tilde{i}_{1:N_P-1}, i_N)}_{(N_P|k+1)} = \phi^{(i_N)}\hat{\delta}^{\tilde{i}_{1:N_P-1}}_{(N_P-1|k+1)}.$$

Similarly, define the optimal state $x^*_{(\tau|k)}$ and input set $u^*_{(\tau|k)}$ at time k as

$$x^*_{(\tau|k)} \in \mathcal{X}^*_{(\tau|k)} = Co\{x^{(\tilde{i}_{1:\tau})*}_{(\tau|k)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}$$
$$u^*_{(\tau|k)} \in \mathcal{U}^*_{(\tau|k)} = Co\{u^{(\tilde{i}_{1:\tau})*}_{(\tau|k)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}$$

By using the optimal solution at time k, the state set $\hat{\mathcal{X}}_{(\tau|k+1)}$ and input set $\hat{\mathcal{U}}_{(\tau|k+1)}$ at time $k+1$ can be constructed as:

$$\hat{\mathcal{X}}_{(\tau|k+1)} = Co\{\hat{x}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}, \tag{24}$$
$$\hat{\mathcal{U}}_{(\tau|k+1)} = Co\{\hat{u}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)}, \tilde{i}_{1:\tau} \in \mathcal{L}_\tau\}, \tag{25}$$

constructing the vertices as follows:

$$\begin{aligned}
\hat{x}_{(0|k+1)} &= \sum_{i=1}^{I} h^{(i)} x^{(i)*}_{(1|k)} + w^{(i)*}_{(1|k)},\\
\hat{x}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)} &= \sum_{i=1}^{L} h^{(i)} x^{(i,\tilde{i}_{1:\tau})*}_{(\tau+1|k)} + \phi^{(i_\tau)} \cdots \phi^{(i_1)} w^*_{(1|k)},\\
&\quad \tau \in \mathbb{N}_{[1,N_P-1]},\\
\hat{x}^{(\tilde{i}_{1:N_P-1}, i_{N_P})}_{(N_P|k+1)} &= \phi^{iN} \hat{x}^{\tilde{i}_{1:N_P-1}}_{(N_P-1|k+1)},
\end{aligned}$$

$$\begin{aligned}
\hat{u}_{(0|k+1)} &= \sum_{i=1}^{I} h^{(i)} u^{(i)*}_{(1|k)} + K w^{(i)*}_{(1|k)},\\
\hat{u}^{\tilde{i}_{1:\tau}}_{(\tau|k+1)} &= \sum_{i=1}^{L} h^{(i)} u^{(i,\tilde{i}_{1:\tau})*}_{(\tau+1|k)} + \phi^{(i_\tau)} \cdots \phi^{(i_1)} w^*_{(1|k)},\\
&\quad \tau \in \mathbb{N}_{[1,N_P-2]},\\
\hat{u}^{\tilde{i}_{1:N_P-1}}_{(N_P-1|k+1)} &= K \hat{x}^{\tilde{i}_{1:N_P-1}}_{(N_P-1|k+1)}.
\end{aligned}$$

Thus, the vertices constructed above satisfy the constraints at time $k+1$. Therefore, the constructed solution $\hat{\sigma}(k+1), \hat{l}(k+1)$ can be solved at time $k+1$.

The stability of the Algorithm 1 is declared now. Define the optimal objective function at time k as $V(\lambda^*(k)) \triangleq (\lambda^*(k))^T N(k) \lambda^*(k)$ and at time $k+1$ as $V(\hat{\lambda}(k+1)) \triangleq (\hat{\lambda}(k+1))^T N(k+1) \hat{\lambda}(k+1)$. At time k, the optimal augmented vector is defined as $\theta^*_{(\tau|k)}$ and is given by the following expression:

$$\begin{aligned}
\theta^*_{(\tau|k)} = col[&\underline{\gamma}^*_{(\tau|k)}, \underline{\upsilon}^*_{(\tau|k)}, \delta^*_{(\tau|k)},\\
&\underline{\sigma}^*_{(\tau|k)}, \underline{H}^*_{(\tau|k)}],
\end{aligned} \tag{26}$$

The augmented vector at time $k+1$ is defined as $\hat{\theta}_{(\tau|k+1)}$.

$$\hat{\theta}_{(\tau|k+1)} = col[\underline{\hat{\gamma}}_{(\tau|k+1)}, \underline{\hat{\upsilon}}_{(\tau|k+1)}, \underline{\hat{\delta}}_{(\tau|k+1)}, \tag{27}$$
$$\underline{\hat{\sigma}}_{(\tau|k+1)}, \underline{\hat{H}}(\tau|k+1)]. \tag{28}$$

Given that $\lambda^*_{(0|k)} = D(k)(\theta^*_{(0|k)})$ and $\hat{\lambda}_{(0|k+1)} = D(k+1)\hat{\theta}_{(0|k+1)}$ are valid, hence the optimal objective function at time k and time $k+1$ is defined as:

$$\begin{aligned}
V(\lambda^*_{(0|k)}) &= (\theta^*_{(0|k)})^T P \theta^*_{(0|k)},\\
V(\hat{\lambda}_{(0|k+1)}) &= (\hat{\theta}_{(0|k+1)})^T P \hat{\theta}_{(0|k+1)}.
\end{aligned} \tag{29}$$

It can be obtained that $\theta_{(0|k+1)} = \theta^*_{(1|k)} = \zeta\theta^*_{(0|k)} + d^*_{(1|k)}$, hence

$$\begin{aligned}
&V(\lambda^*_{(0|k)}) - V(\hat{\lambda}_{(0|k+1)})\\
&\geq \left\|x_{(0|k)}\right\|_Q^2 + \left\|u_{(0|k)}\right\|_R^2 - n\bar{d}.
\end{aligned} \tag{30}$$

where $n > 0$, $\bar{d} = \max_{d \in \mathbb{D}} \|d\|_2$. The system is asymptotically stable. ∎

IV. SIMULATION

In this section, a simulation example is given to verify the efficacy of the data selection method. The data selection method using controller performance benchmark can be applied in input-mapping data-driven MPC with uncertain models. For brevity, it is named input-mapping data selection with controller performance method(IM-DS-CPM). Consider that the parameters of the uncertain system (1) are as follows:

$$A0 = \begin{bmatrix} 0.8623 & 0 & 0.1472 & 0\\ 0 & 0.8359 & 0 & 0.1052\\ 0 & 0 & 0.6434 & 0\\ 0 & 0 & 0 & 0.8130 \end{bmatrix},$$

$$B0 = \begin{bmatrix} 0.0647 & 0\\ 0 & 0.0647\\ 0 & 0.0680\\ 0.0619 & 0 \end{bmatrix},$$

$$\Delta_A^{(1)} = \begin{bmatrix} 0 & 0 & -0.1177 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0.3163 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta_B^{(1)} = \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & -0.0204\\ 0 & 0 \end{bmatrix},$$

$$\Delta_A^{(2)} = -\Delta_A^{(1)},\ \Delta_B^{(2)} = -\Delta_A^{(1)}.$$

The disturbance $\|d\|_\infty \leq 0.01$, state constraints are $\|x\|_\infty \leq 15$, input constraints are $\|u\|_\infty \leq 15$.

Set the length of historical data M as 6 and the length of the sliding window N_S as 7. Select an optimal time domain length N_P as 4. The threshold value δ_x is set as 1. The initial state is set as $x_0 = [9, 11, 7.5, 8]^T$, and at the time 70, the system state is reset to its initial condition. The error weight matrix Q is $diag\{2, 0.1, 6, 0.1\}$ and the input weight matrix R is $0.01I_2$. IM-DS-CPM is compared with the method in [7], which can be shorted as input-mapping MPC (IM), and the method in [9], which used a similarity index to check the similarity between the current data and the past data items. For brevity, the method [9] is named as input-mapping data selection with similarity(IM-DS-SIM).

The results can be seen in Fig.2-5. Upon examination of Fig.4, where the state x_3 is assigned the greatest weight, it is evident that the data selection can accelerate the system. Consequently, the data-driven selection strategy we have proposed is adept at identifying the most informative data and improving the performance of the controller.

V. CONCLUSION

This paper proposes a new data selection strategy for data-driven MPC with uncertain models. The input-mapping MPC approach uses historical data to linearly represent the system's

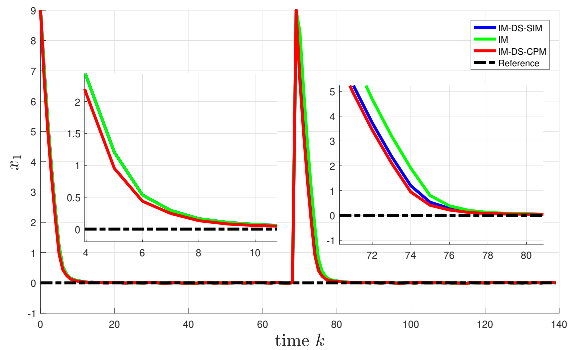

Fig. 2. The trajectories of state $x_1(k)$.

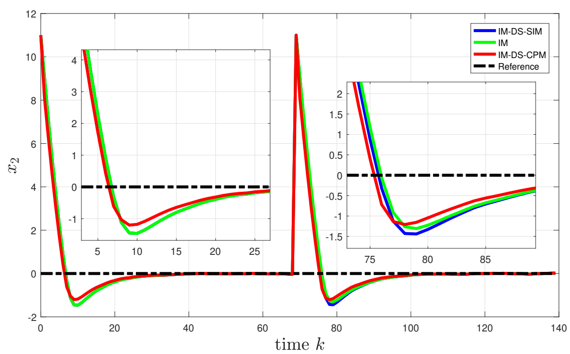

Fig. 3. The trajectories of state $x_2(k)$.

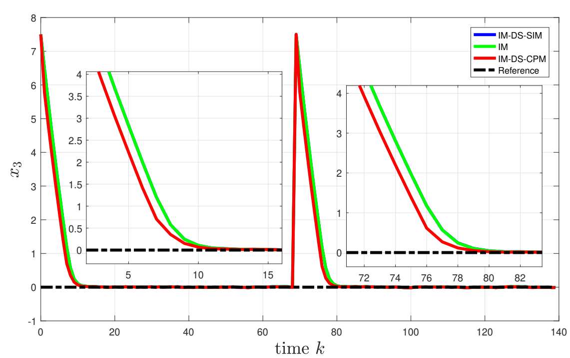

Fig. 4. The trajectories of state $x_3(k)$.

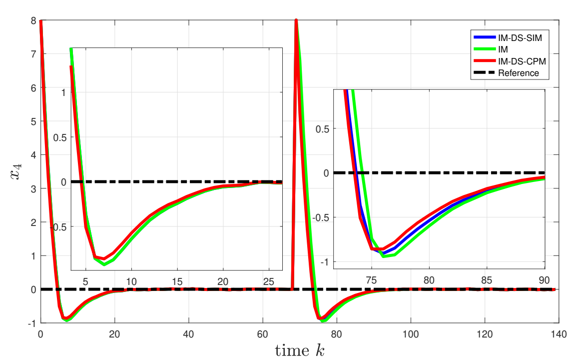

Fig. 5. The trajectories of state $x_4(k)$.

current and future states and control inputs. To enhance controller performance, a data-driven framework is introduced to evaluate the controller's effectiveness. Key to this method is a sliding window mechanism, which identifies data points that most improve performance. The optimal combination coefficients of historical data are derived through solving an optimization problem. The stability of the resulting closed-loop system is analyzed. The proposed method's validity is demonstrated through an example for an uncertain system.

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